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a. $3x^2 - 2x + 8 = 0$ b. $3x^2 - 2x - 8 = 0$

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Solving 2nd Degree Equations $ax^2 + bx + c = 0$

Example D. Solve $3x^2 - 2x + 2 = 0$

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The complex solutions are

$$x = \frac{2 \pm \sqrt{-20}}{6}$$

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1st and 2nd degree equations are the most important equations in science and now we know how to solve all of them efficiently.

Applications of higher degree equations are more specialized and in general we solve them using computers.

Solving 2nd Degree Equations $ax^2 + bx + c = 0$

Exercise A. Solve. Use the square root methods. Give both the exact answers and the numerical answers. If the answer is not real, state so.

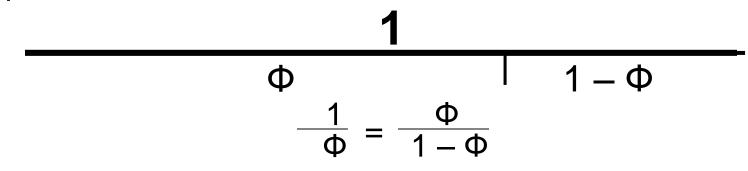
2. $3x^2 - 15 = 0$ 1. $3x^2 - 12 = 0$ 3. $3x^2 + 15 = 0$ 4. $x^2 - 3 = -x^2 + 15$ 5. $-6 = 4x^2 - 15$ 6. $(x-2)^2 = 2$ 7. 4 = $(2x - 3)^2$ Exercise B. Solve by factoring and by the quadratic formula. 8. $x^2 - 3x = 4$ 9. x - 15 = 2x10. $x^2 + 5x + 12 = 0$ 11. $-x^2 - 2x + 8 = 0$ 12. $9 - 3x - 2x^2 = 0$ 13. $2x^2 - x - 1 = 0$ 15. x(x - 2) = 24 16. $2x^2 = 3(x + 1) - 1$ 14. $x^2 - 3x = 10$ Exercise C. Solve by the quadratic formula. If the answers are not real numbers, state so. 17. $x^2 - x + 1 = 0$ 18. $x^2 - x - 1 = 0$ 19. $x^2 - 3x - 2 = 0$ 20. $x^2 - 2x + 3 = 0$ 21. $2x^2 - 3x - 1 = 0$ 22. $3x^2 = 2x + 3$

Solving 2nd Degree Equations $ax^2 + bx + c = 0$ Exercise C. Solve by the quadratic formula. If the answers are not real numbers, state so.

23.
$$2(x^2 - 1) + x = 4$$

24. $(x - 1)(x + 1) = 2x(x + 2)$
25. $\frac{(x - 1)}{(x + 2)} = \frac{2x}{(x + 2)}$
26. $\frac{(x + 1)}{(x + 2)} = \frac{(x + 2)}{(2x + 1)}$

27. Cut a stick of unit length (1) into two parts in a manner such that "the ratio of the whole to the large part is the same as the ratio of the large part to the small part" In picture,



the golden ratio Φ (phi) is the ratio 1 : Φ . Find the exact and approximate value of Φ . (Google "golden ratio")